

## DIFFERENTIATION RULES

$$\frac{d}{dx}(f(x) + g(x)) = f'(x) + g'(x)$$

$$\frac{d}{dx}\left(\frac{1}{f(x)}\right) = -\frac{f'(x)}{(f(x))^2}$$

$$\frac{d}{dx}(cf(x)) = cf'(x)$$

$$\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{g(x)f'(x) - f(x)g'(x)}{(g(x))^2}$$

$$\frac{d}{dx}(f(x)g(x)) = f'(x)g(x) + f(x)g'(x)$$

$$\frac{d}{dx}f(g(x)) = f'(g(x))g'(x)$$

## ELEMENTARY DERIVATIVES

$$\frac{d}{dx}\frac{1}{x} = -\frac{1}{x^2}$$

$$\frac{d}{dx}e^x = e^x$$

$$\frac{d}{dx}\sin x = \cos x$$

$$\frac{d}{dx}\sec x = \sec x \tan x$$

$$\frac{d}{dx}\sin^{-1}x = \frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}\sqrt{x} = \frac{1}{2\sqrt{x}}$$

$$\frac{d}{dx}a^x = a^x \ln a \quad (a > 0)$$

$$\frac{d}{dx}\cos x = -\sin x$$

$$\frac{d}{dx}\csc x = -\csc x \cot x$$

$$\frac{d}{dx}\tan^{-1}x = \frac{1}{1+x^2}$$

$$\frac{d}{dx}x^r = rx^{r-1}$$

$$\frac{d}{dx}\ln x = \frac{1}{x} \quad (x > 0)$$

$$\frac{d}{dx}\tan x = \sec^2 x$$

$$\frac{d}{dx}\cot x = -\csc^2 x$$

$$\frac{d}{dx}|x| = \operatorname{sgn} x = \frac{x}{|x|}$$

## TRIGONOMETRIC IDENTITIES

$$\sin^2 x + \cos^2 x = 1$$

$$\sec^2 x = 1 + \tan^2 x$$

$$\csc^2 x = 1 + \cot^2 x$$

$$\sin(x \pm y) = \sin x \cos y \pm \cos x \sin y$$

$$\sin 2x = 2 \sin x \cos x$$

$$\cos 2x = 2 \cos^2 x - 1 = 1 - 2 \sin^2 x$$

$$\sin(-x) = -\sin x$$

$$\sin(\pi - x) = \sin x$$

$$\sin\left(\frac{\pi}{2} - x\right) = \cos x$$

$$\cos(x \pm y) = \cos x \cos y \mp \sin x \sin y$$

$$\sin^2 x = \frac{1 - \cos 2x}{2}$$

$$\cos(-x) = \cos x$$

$$\cos(\pi - x) = -\cos x$$

$$\cos\left(\frac{\pi}{2} - x\right) = \sin x$$

$$\tan(x \pm y) = \frac{\tan x \pm \tan y}{1 \mp \tan x \tan y}$$

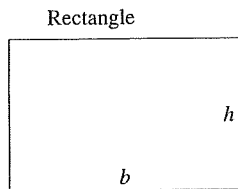
$$\cos^2 x = \frac{1 + \cos 2x}{2}$$

## QUADRATIC FORMULA

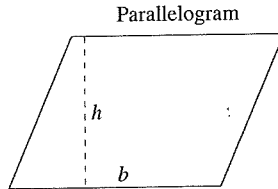
If  $Ax^2 + Bx + C = 0$ , then  $x = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$ .

## GEOMETRIC FORMULAS

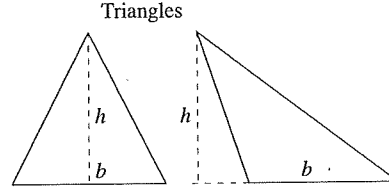
$A$  = area,  
 $b$  = base,  
 $h$  = height,  
 $C$  = circumference,  $V$  = volume,  
 $S$  = surface area



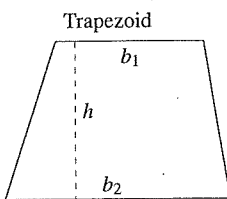
$$A = bh$$



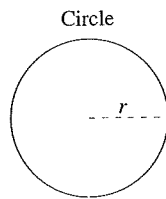
$$A = bh$$



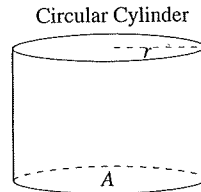
$$A = \frac{1}{2}bh$$



$$A = \frac{1}{2}(b_1 + b_2)h$$

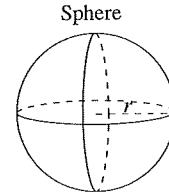


$$A = \pi r^2, \quad C = 2\pi r$$

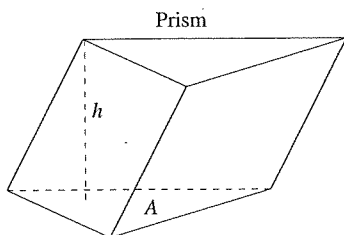


$$V = Ah = \pi r^2 h, \quad S = Ch = 2\pi r h$$

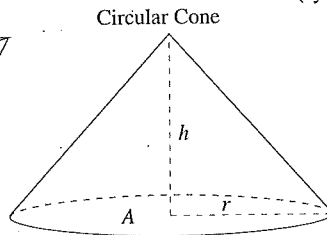
(cylindrical wall)



$$V = \frac{4}{3}\pi r^3, \quad S = 4\pi r^2$$

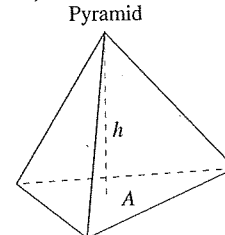


$$V = Ah$$



$$V = \frac{1}{3}Ah = \frac{1}{3}\pi r^2 h, \quad S = \pi r \sqrt{r^2 + h^2}$$

(conical wall)



$$V = \frac{1}{3}Ah$$

## VECTOR IDENTITIES

If  $\mathbf{u} = u_1\mathbf{i} + u_2\mathbf{j} + u_3\mathbf{k}$  then (dot product)  $\mathbf{u} \cdot \mathbf{v} = u_1v_1 + u_2v_2 + u_3v_3$

$$\mathbf{v} = v_1\mathbf{i} + v_2\mathbf{j} + v_3\mathbf{k}$$

$$\mathbf{w} = w_1\mathbf{i} + w_2\mathbf{j} + w_3\mathbf{k}$$

(cross product)

$$\mathbf{u} \times \mathbf{v} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \end{vmatrix} = (u_2v_3 - u_3v_2)\mathbf{i} + (u_3v_1 - u_1v_3)\mathbf{j} + (u_1v_2 - u_2v_1)\mathbf{k}$$

$$\text{length of } \mathbf{u} = |\mathbf{u}| = \sqrt{\mathbf{u} \cdot \mathbf{u}} = \sqrt{u_1^2 + u_2^2 + u_3^2}$$

$$\text{angle between } \mathbf{u} \text{ and } \mathbf{v} = \cos^{-1} \left( \frac{\mathbf{u} \cdot \mathbf{v}}{|\mathbf{u}||\mathbf{v}|} \right)$$

triple product identities

$$\mathbf{u} \cdot (\mathbf{v} \times \mathbf{w}) = \mathbf{v} \cdot (\mathbf{w} \times \mathbf{u}) = \mathbf{w} \cdot (\mathbf{u} \times \mathbf{v})$$

$$\mathbf{u} \times (\mathbf{v} \times \mathbf{w}) = (\mathbf{u} \cdot \mathbf{w})\mathbf{v} - (\mathbf{u} \cdot \mathbf{v})\mathbf{w}$$

## IDENTITIES INVOLVING GRADIENT, DIVERGENCE, CURL, AND LAPLACIAN

$\nabla = \mathbf{i} \frac{\partial}{\partial x} + \mathbf{j} \frac{\partial}{\partial y} + \mathbf{k} \frac{\partial}{\partial z}$  ("del" or "nabla" operator)

$$\mathbf{F}(x, y, z) = F_1(x, y, z)\mathbf{i} + F_2(x, y, z)\mathbf{j} + F_3(x, y, z)\mathbf{k}$$

$$\nabla \phi(x, y, z) = \mathbf{grad} \phi(x, y, z) = \frac{\partial \phi}{\partial x}\mathbf{i} + \frac{\partial \phi}{\partial y}\mathbf{j} + \frac{\partial \phi}{\partial z}\mathbf{k}$$

$$\nabla \cdot \mathbf{F}(x, y, z) = \text{div } \mathbf{F}(x, y, z) = \frac{\partial F_1}{\partial x} + \frac{\partial F_2}{\partial y} + \frac{\partial F_3}{\partial z}$$

$$\begin{aligned} \nabla \times \mathbf{F}(x, y, z) &= \mathbf{curl} \mathbf{F}(x, y, z) = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ F_1 & F_2 & F_3 \end{vmatrix} \\ &= \left( \frac{\partial F_3}{\partial y} - \frac{\partial F_2}{\partial z} \right)\mathbf{i} + \left( \frac{\partial F_1}{\partial z} - \frac{\partial F_3}{\partial x} \right)\mathbf{j} + \left( \frac{\partial F_2}{\partial x} - \frac{\partial F_1}{\partial y} \right)\mathbf{k} \end{aligned}$$

$$\nabla(\phi\psi) = \phi\nabla\psi + \psi\nabla\phi$$

$$\nabla \cdot (\mathbf{F} \times \mathbf{G}) = (\nabla \times \mathbf{F}) \cdot \mathbf{G} - \mathbf{F} \cdot (\nabla \times \mathbf{G})$$

$$\nabla \cdot (\phi\mathbf{F}) = (\nabla\phi) \cdot \mathbf{F} + \phi(\nabla \cdot \mathbf{F})$$

$$\nabla \times (\mathbf{F} \times \mathbf{G}) = \mathbf{F}(\nabla \cdot \mathbf{G}) - \mathbf{G}(\nabla \cdot \mathbf{F}) - (\mathbf{F} \cdot \nabla)\mathbf{G} + (\mathbf{G} \cdot \nabla)\mathbf{F}$$

$$\nabla \times (\phi\mathbf{F}) = (\nabla\phi) \times \mathbf{F} + \phi(\nabla \times \mathbf{F})$$

$$\nabla(\mathbf{F} \cdot \mathbf{G}) = \mathbf{F} \times (\nabla \times \mathbf{G}) + \mathbf{G} \times (\nabla \times \mathbf{F}) + (\mathbf{F} \cdot \nabla)\mathbf{G} + (\mathbf{G} \cdot \nabla)\mathbf{F}$$

$$\nabla \times (\nabla\phi) = \mathbf{0} \quad (\mathbf{curl} \mathbf{grad} = \mathbf{0})$$

$$\nabla \cdot (\nabla \times \mathbf{F}) = \mathbf{0} \quad (\mathbf{div} \mathbf{curl} = \mathbf{0})$$

$$\nabla^2 \phi(x, y, z) = \nabla \cdot \nabla \phi(x, y, z) = \text{div } \mathbf{grad} \phi = \frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} + \frac{\partial^2 \phi}{\partial z^2}$$

$$\nabla \times (\nabla \times \mathbf{F}) = \nabla(\nabla \cdot \mathbf{F}) - \nabla^2 \mathbf{F} \quad (\mathbf{curl} \mathbf{curl} = \mathbf{grad} \mathbf{div} - \mathbf{laplacian})$$

## VERSIONS OF THE FUNDAMENTAL THEOREM OF CALCULUS

$$\int_a^b f'(t) dt = f(b) - f(a) \quad (\text{the one-dimensional Fundamental Theorem})$$

$$\int_C \mathbf{grad} \phi \cdot d\mathbf{r} = \phi(\mathbf{r}(b)) - \phi(\mathbf{r}(a)) \text{ if } C \text{ is the curve } \mathbf{r} = \mathbf{r}(t), (a \leq t \leq b).$$

$$\iint_R \left( \frac{\partial F_2}{\partial x} - \frac{\partial F_1}{\partial y} \right) dA = \oint_C \mathbf{F} \cdot d\mathbf{r} = \oint_C F_1(x, y) dx + F_2(x, y) dy \text{ where } C \text{ is the positively oriented boundary of } R \quad (\text{Green's Theorem})$$

$$\iint_S \mathbf{curl} \mathbf{F} \cdot \hat{\mathbf{N}} dS = \oint_C \mathbf{F} \cdot d\mathbf{r} = \oint_C F_1(x, y, z) dx + F_2(x, y, z) dy + F_3(x, y, z) dz \text{ where } C \text{ is the oriented boundary of } S. \quad (\text{Stokes's Theorem})$$

Three-dimensional versions:  $S$  is the closed boundary of  $D$ , with outward normal  $\hat{\mathbf{N}}$

$$\iiint_D \text{div } \mathbf{F} dV = \iint_S \mathbf{F} \cdot \hat{\mathbf{N}} dS \quad \text{Divergence Theorem} \quad \iiint_D \mathbf{curl} \mathbf{F} dV = - \iint_S \mathbf{F} \times \hat{\mathbf{N}} dS$$

$$\iiint_D \mathbf{grad} \phi dV = \iint_S \phi \hat{\mathbf{N}} dS$$

## FORMULAS RELATING TO CURVES IN 3-SPACE

Curve:  $\mathbf{r} = \mathbf{r}(t) = x(t)\mathbf{i} + y(t)\mathbf{j} + z(t)\mathbf{k}$

$$\text{Velocity: } \mathbf{v} = \frac{d\mathbf{r}}{dt} = v\hat{\mathbf{T}}$$

$$\text{Speed: } v = |\mathbf{v}| = \frac{ds}{dt}$$

Arc length:  $s = \int_{t_0}^t v dt$

$$\text{Acceleration: } \mathbf{a} = \frac{d\mathbf{v}}{dt} = \frac{d^2\mathbf{r}}{dt^2}$$

$$\text{Tangential and normal components: } \mathbf{a} = \frac{dv}{dt}\hat{\mathbf{T}} + v^2\kappa\hat{\mathbf{N}}$$

Unit tangent:  $\hat{\mathbf{T}} = \frac{\mathbf{v}}{v}$

$$\text{Binormal: } \hat{\mathbf{B}} = \frac{\mathbf{v} \times \mathbf{a}}{|\mathbf{v} \times \mathbf{a}|}$$

$$\text{Normal: } \hat{\mathbf{N}} = \hat{\mathbf{B}} \times \hat{\mathbf{T}} = \frac{d\hat{\mathbf{T}}/dt}{|d\hat{\mathbf{T}}/dt|}$$

Curvature:  $\kappa = \frac{|\mathbf{v} \times \mathbf{a}|}{v^3}$

$$\text{Radius of curvature: } \rho = \frac{1}{\kappa}$$

$$\text{Torsion: } \tau = \frac{(\mathbf{v} \times \mathbf{a}) \cdot (d\mathbf{a}/dt)}{|\mathbf{v} \times \mathbf{a}|^2}$$

$$\text{The Frenet-Serret formulas: } \frac{d\hat{\mathbf{T}}}{ds} = \kappa\hat{\mathbf{N}}, \quad \frac{d\hat{\mathbf{N}}}{ds} = -\kappa\hat{\mathbf{T}} + \tau\hat{\mathbf{B}}, \quad \frac{d\hat{\mathbf{B}}}{ds} = -\tau\hat{\mathbf{N}}$$

## MISCELLANEOUS ALGEBRAIC INTEGRALS

$$\int x(ax+b)^{-1} dx = \frac{x}{a} - \frac{b}{a^2} \ln|ax+b| + C$$

$$\int x(ax+b)^{-2} dx = \frac{1}{a^2} \left[ \ln|ax+b| + \frac{b}{ax+b} \right] + C$$

$$\int x(ax+b)^n dx = \frac{(ax+b)^{n+1}}{a^2} \left( \frac{ax+b}{n+2} - \frac{b}{n+1} \right) + C \text{ if } n \neq -1, -2$$

$$\int \frac{dx}{(a^2 \pm x^2)^n} = \frac{1}{2a^2(n-1)} \left( \frac{x}{(a^2 \pm x^2)^{n-1}} + (2n-3) \int \frac{dx}{(a^2 \pm x^2)^{n-1}} \right) \text{ if } n \neq 1$$

$$\int x\sqrt{ax+b} dx = \frac{2}{15a^2} (3ax-2b)(ax+b)^{3/2} + C$$

$$\int x^n \sqrt{ax+b} dx = \frac{2}{a(2n+3)} \left( x^n(ax+b)^{3/2} - nb \int x^{n-1} \sqrt{ax+b} dx \right)$$

$$\int \frac{x dx}{\sqrt{ax+b}} = \frac{2}{3a^2} (ax-2b)\sqrt{ax+b} + C$$

$$\int \frac{x^n dx}{\sqrt{ax+b}} = \frac{2}{a(2n+1)} \left( x^n \sqrt{ax+b} - nb \int \frac{x^{n-1}}{\sqrt{ax+b}} dx \right)$$

$$\int \frac{dx}{x\sqrt{ax+b}} = \frac{1}{\sqrt{b}} \ln \left| \frac{\sqrt{ax+b} - \sqrt{b}}{\sqrt{ax+b} + \sqrt{b}} \right| + C \text{ if } b > 0$$

$$\int \frac{dx}{x\sqrt{ax+b}} = \frac{2}{\sqrt{-b}} \tan^{-1} \sqrt{\frac{ax+b}{-b}} + C \text{ if } b < 0$$

$$\int \frac{dx}{x^n \sqrt{ax+b}} = -\frac{\sqrt{ax+b}}{b(n-1)x^{n-1}} - \frac{(2n-3)a}{(2n-2)b} \int \frac{dx}{x^{n-1} \sqrt{ax+b}} \text{ if } n \neq 1$$

$$\int \sqrt{2ax-x^2} dx = \frac{x-a}{2} \sqrt{2ax-x^2} + \frac{a^2}{2} \sin^{-1} \frac{x-a}{a} + C \quad (a > 0)$$

$$\int \frac{dx}{\sqrt{2ax-x^2}} = \sin^{-1} \frac{x-a}{a} + C \quad (a > 0)$$

$$\int x^n \sqrt{2ax-x^2} dx = -\frac{x^{n-1}(2ax-x^2)^{3/2}}{n+2} + \frac{(2n+1)a}{n+2} \int x^{n-1} \sqrt{2ax-x^2} dx$$

$$\int \frac{x^n dx}{\sqrt{2ax-x^2}} = -\frac{x^{n-1}}{n} \sqrt{2ax-x^2} + \frac{(2n-1)a}{n} \int \frac{x^{n-1} dx}{\sqrt{2ax-x^2}}$$

$$\int \frac{\sqrt{2ax-x^2}}{x} dx = \sqrt{2ax-x^2} + a \sin^{-1} \frac{x-a}{a} + C \quad (a > 0)$$

$$\int \frac{\sqrt{2ax-x^2}}{x^n} dx = \frac{(2ax-x^2)^{3/2}}{(3-2n)ax^n} + \frac{n-3}{(2n-3)a} \int \frac{\sqrt{2ax-x^2}}{x^{n-1}} dx$$

$$\int \frac{dx}{x^n \sqrt{2ax-x^2}} = \frac{\sqrt{2ax-x^2}}{a(1-2n)x^n} + \frac{n-1}{(2n-1)a} \int \frac{dx}{x^{n-1} \sqrt{2ax-x^2}}$$

$$\int (\sqrt{2ax-x^2})^n dx = \frac{x-a}{n+1} (\sqrt{2ax-x^2})^n + \frac{na^2}{n+1} \int (\sqrt{2ax-x^2})^{n-2} dx \text{ if } n \neq -1$$

$$\int \frac{dx}{(\sqrt{2ax-x^2})^n} = \frac{x-a}{(n-2)a^2} (\sqrt{2ax-x^2})^{2-n} + \frac{n-3}{(n-2)a^2} \int \frac{dx}{(\sqrt{2ax-x^2})^{n-2}} \text{ if } n \neq 2$$

## DEFINITE INTEGRALS

$$\int_0^\infty x^n e^{-x} dx = n! \quad (n \geq 0)$$

$$\int_0^\infty e^{-ax^2} dx = \frac{1}{2} \sqrt{\frac{\pi}{a}} \quad a > 0$$

$$\int_0^\infty x e^{-ax^2} dx = \frac{1}{2a} \text{ if } a > 0$$

$$\int_0^\infty x^n e^{-ax^2} dx = \frac{n-1}{2a} \int_0^\infty x^{n-2} e^{-ax^2} dx \text{ if } a > 0, n \geq 2$$

$$\int_0^{\pi/2} \sin^n x dx = \int_0^{\pi/2} \cos^n x dx = \begin{cases} \frac{1 \cdot 3 \cdot 5 \cdots (n-1)}{2 \cdot 4 \cdot 6 \cdots n} \frac{\pi}{2} & \text{if } n \text{ is an even integer and } n \geq 2 \\ \frac{2 \cdot 4 \cdot 6 \cdots (n-1)}{3 \cdot 5 \cdot 7 \cdots n} & \text{if } n \text{ is an odd integer and } n \geq 3 \end{cases}$$

**INTEGRALS INVOLVING  $\sqrt{x^2 \pm a^2}$  ( $a > 0$ )**(If  $\sqrt{x^2 - a^2}$ , assume  $x > a > 0$ .)

$$\int \sqrt{x^2 \pm a^2} dx = \frac{x}{2} \sqrt{x^2 \pm a^2} \pm \frac{a^2}{2} \ln|x + \sqrt{x^2 \pm a^2}| + C$$

$$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln|x + \sqrt{x^2 \pm a^2}| + C$$

$$\int \frac{\sqrt{x^2 + a^2}}{x} dx = \sqrt{x^2 + a^2} - a \ln \left| \frac{a + \sqrt{x^2 + a^2}}{x} \right| + C$$

$$\int \frac{\sqrt{x^2 - a^2}}{x} dx = \sqrt{x^2 - a^2} - a \tan^{-1} \frac{\sqrt{x^2 - a^2}}{a} + C$$

$$\int x^2 \sqrt{x^2 \pm a^2} dx = \frac{x}{8} (2x^2 \pm a^2) \sqrt{x^2 \pm a^2} - \frac{a^4}{8} \ln|x + \sqrt{x^2 \pm a^2}| + C$$

$$\int \frac{x^2}{\sqrt{x^2 \pm a^2}} dx = \frac{x}{2} \sqrt{x^2 \pm a^2} \mp \frac{a^2}{2} \ln|x + \sqrt{x^2 \pm a^2}| + C$$

$$\int \frac{\sqrt{x^2 \pm a^2}}{x^2} dx = -\frac{\sqrt{x^2 \pm a^2}}{x} + \ln|x + \sqrt{x^2 \pm a^2}| + C$$

$$\int \frac{dx}{x^2 \sqrt{x^2 \pm a^2}} = \mp \frac{\sqrt{x^2 \pm a^2}}{a^2 x} + C$$

$$\int \frac{dx}{(x^2 \pm a^2)^{3/2}} = \frac{\pm x}{a^2 \sqrt{x^2 \pm a^2}} + C$$

$$\int (x^2 \pm a^2)^{3/2} dx = \frac{x}{8} (2x^2 \pm 5a^2) \sqrt{x^2 \pm a^2} + \frac{3a^4}{8} \ln|x + \sqrt{x^2 \pm a^2}| + C$$

**INTEGRALS INVOLVING  $\sqrt{a^2 - x^2}$  ( $a > 0, |x| < a$ )**

$$\int \sqrt{a^2 - x^2} dx = \frac{x}{2} \sqrt{a^2 - x^2} + \frac{a^2}{2} \sin^{-1} \frac{x}{a} + C$$

$$\int \frac{\sqrt{a^2 - x^2}}{x} dx = \sqrt{a^2 - x^2} - a \ln \left| \frac{a + \sqrt{a^2 - x^2}}{x} \right| + C$$

$$\int \frac{x^2}{\sqrt{a^2 - x^2}} dx = -\frac{x}{2} \sqrt{a^2 - x^2} + \frac{a^2}{2} \sin^{-1} \frac{x}{a} + C$$

$$\int x^2 \sqrt{a^2 - x^2} dx = \frac{x}{8} (2x^2 - a^2) \sqrt{a^2 - x^2} + \frac{a^4}{8} \sin^{-1} \frac{x}{a} + C$$

$$\int \frac{dx}{x^2 \sqrt{a^2 - x^2}} = -\frac{\sqrt{a^2 - x^2}}{a^2 x} + C$$

$$\int \frac{\sqrt{a^2 - x^2}}{x^2} dx = -\frac{\sqrt{a^2 - x^2}}{x} - \sin^{-1} \frac{x}{a} + C$$

$$\int \frac{dx}{x \sqrt{a^2 - x^2}} = -\frac{1}{a} \ln \left| \frac{a + \sqrt{a^2 - x^2}}{x} \right| + C$$

$$\int \frac{dx}{(a^2 - x^2)^{3/2}} = \frac{x}{a^2 \sqrt{a^2 - x^2}} + C$$

$$\int (a^2 - x^2)^{3/2} dx = \frac{x}{8} (5a^2 - 2x^2) \sqrt{a^2 - x^2} + \frac{3a^4}{8} \sin^{-1} \frac{x}{a} + C$$

**INTEGRALS OF INVERSE TRIGONOMETRIC FUNCTIONS**

$$\int \sin^{-1} x dx = x \sin^{-1} x + \sqrt{1 - x^2} + C$$

$$\int \tan^{-1} x dx = x \tan^{-1} x - \frac{1}{2} \ln(1 + x^2) + C$$

$$\int \sec^{-1} x dx = x \sec^{-1} x - \ln|x + \sqrt{x^2 - 1}| + C \quad (x > 1)$$

$$\int x \sin^{-1} x dx = \frac{1}{4} (2x^2 - 1) \sin^{-1} x + \frac{x}{4} \sqrt{1 - x^2} + C$$

$$\int x \tan^{-1} x dx = \frac{1}{2} (x^2 + 1) \tan^{-1} x - \frac{x}{2} + C$$

$$\int x \sec^{-1} x dx = \frac{x^2}{2} \sec^{-1} x - \frac{1}{2} \sqrt{x^2 - 1} + C \quad (x > 1)$$

$$\int x^n \sin^{-1} x dx = \frac{x^{n+1}}{n+1} \sin^{-1} x - \frac{1}{n+1} \int \frac{x^{n+1}}{\sqrt{1-x^2}} dx + C \text{ if } n \neq -1$$

$$\int x^n \tan^{-1} x dx = \frac{x^{n+1}}{n+1} \tan^{-1} x - \frac{1}{n+1} \int \frac{x^{n+1}}{1+x^2} dx + C \text{ if } n \neq -1$$

$$\int x^n \sec^{-1} x dx = \frac{x^{n+1}}{n+1} \sec^{-1} x - \frac{1}{n+1} \int \frac{x^n}{\sqrt{x^2-1}} dx + C \quad (n \neq -1, x > 1)$$

**EXPONENTIAL AND LOGARITHMIC INTEGRALS**

$$\int x e^x dx = (x - 1) e^x + C$$

$$\int x^n e^x dx = x^n e^x - n \int x^{n-1} e^x dx$$

$$\int \ln x dx = x \ln x - x + C$$

$$\int x^n \ln x dx = \frac{x^{n+1}}{n+1} \ln x - \frac{x^{n+1}}{(n+1)^2} + C, \quad (n \neq -1)$$

$$\int x^n (\ln x)^m dx = \frac{x^{n+1}}{n+1} (\ln x)^m - \frac{m}{n+1} \int x^n (\ln x)^{m-1} dx \quad (n \neq -1)$$

$$\int e^{ax} \sin bx dx = \frac{e^{ax}}{a^2 + b^2} (a \sin bx - b \cos bx) + C$$

$$\int e^{ax} \cos bx dx = \frac{e^{ax}}{a^2 + b^2} (a \cos bx + b \sin bx) + C$$

**INTEGRALS OF HYPERBOLIC FUNCTIONS**

$$\int \sinh x dx = \cosh x + C$$

$$\int \cosh x dx = \sinh x + C$$

$$\int \tanh x dx = \ln(\cosh x) + C$$

$$\int \coth x dx = \ln|\sinh x| + C$$

$$\int \operatorname{sech} x dx = 2 \tan^{-1}(e^x) + C$$

$$\int \operatorname{csch} x dx = \ln \left| \tanh \frac{x}{2} \right| + C$$

$$\int \sinh^2 x dx = \frac{1}{4} \sinh 2x - \frac{x}{2} + C$$

$$\int \cosh^2 x dx = \frac{1}{4} \sinh 2x + \frac{x}{2} + C$$

$$\int \tanh^2 x dx = x - \tanh x + C$$

$$\int \coth^2 x dx = x - \coth x + C$$

$$\int \operatorname{sech}^2 x dx = \tanh x + C$$

$$\int \operatorname{csch}^2 x dx = -\coth x + C$$

$$\int \operatorname{sech} x \tanh x dx = -\operatorname{sech} x + C$$

$$\int \operatorname{csch} x \coth x dx = -\operatorname{csch} x + C$$

## INTEGRATION RULES

$$\int (Af(x) + Bg(x)) dx = A \int f(x) dx + B \int g(x) dx$$

$$\int f'(g(x)) g'(x) dx = f(g(x)) + C$$

$$\int U(x) dV(x) = U(x)V(x) - \int V(x) dU(x)$$

$$\int_a^b f'(x) dx = f(b) - f(a)$$

$$\frac{d}{dx} \int_a^x f(t) dt = f(x)$$

## ELEMENTARY INTEGRALS

$$\int x^r dx = \frac{1}{r+1} x^{r+1} + C \text{ if } r \neq -1$$

$$\int \frac{dx}{x} = \ln|x| + C$$

$$\int e^x dx = e^x + C$$

$$\int a^x dx = \frac{a^x}{\ln a} + C$$

$$\int \sin x dx = -\cos x + C$$

$$\int \cos x dx = \sin x + C$$

$$\int \sec^2 x dx = \tan x + C$$

$$\int \csc^2 x dx = -\cot x + C$$

$$\int \sec x \tan x dx = \sec x + C$$

$$\int \csc x \cot x dx = -\csc x + C$$

$$\int \tan x dx = \ln|\sec x| + C$$

$$\int \cot x dx = \ln|\sin x| + C$$

$$\int \sec x dx = \ln|\sec x + \tan x| + C$$

$$\int \csc x dx = \ln|\csc x - \cot x| + C$$

$$\int \frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1} \frac{x}{a} + C \quad (a > 0, |x| < a)$$

$$\int \frac{dx}{a^2 + x^2} = \frac{1}{a} \tan^{-1} \frac{x}{a} + C \quad (a > 0)$$

$$\int \frac{dx}{a^2 - x^2} = \frac{1}{2a} \ln \left| \frac{x+a}{x-a} \right| + C \quad (a > 0)$$

$$\int \frac{dx}{x\sqrt{x^2 - a^2}} = \frac{1}{a} \sec^{-1} \left| \frac{x}{a} \right| + C \quad (a > 0, |x| > a)$$

## TRIGONOMETRIC INTEGRALS

$$\int \sin^2 x dx = \frac{x}{2} - \frac{1}{4} \sin 2x + C$$

$$\int \cos^2 x dx = \frac{x}{2} + \frac{1}{4} \sin 2x + C$$

$$\int \tan^2 x dx = \tan x - x + C$$

$$\int \cot^2 x dx = -\cot x - x + C$$

$$\int \sec^3 x dx = \frac{1}{2} \sec x \tan x + \frac{1}{2} \ln|\sec x + \tan x| + C$$

$$\int \csc^3 x dx = -\frac{1}{2} \csc x \cot x + \frac{1}{2} \ln|\csc x - \cot x| + C$$

$$\int \sin ax \sin bx dx = \frac{\sin(a-b)x}{2(a-b)} - \frac{\sin(a+b)x}{2(a+b)} + C \text{ if } a^2 \neq b^2$$

$$\int \cos ax \cos bx dx = \frac{\sin(a-b)x}{2(a-b)} + \frac{\sin(a+b)x}{2(a+b)} + C \text{ if } a^2 \neq b^2$$

$$\int \sin ax \cos bx dx = -\frac{\cos(a-b)x}{2(a-b)} - \frac{\cos(a+b)x}{2(a+b)} + C \text{ if } a^2 \neq b^2$$

$$\int \sin^n x dx = -\frac{1}{n} \sin^{n-1} x \cos x + \frac{n-1}{n} \int \sin^{n-2} x dx$$

$$\int \cos^n x dx = \frac{1}{n} \cos^{n-1} x \sin x + \frac{n-1}{n} \int \cos^{n-2} x dx$$

$$\int \tan^n x dx = \frac{1}{n-1} \tan^{n-1} x - \int \tan^{n-2} x dx \text{ if } n \neq 1$$

$$\int \cot^n x dx = \frac{-1}{n-1} \cot^{n-1} x - \int \cot^{n-2} x dx \text{ if } n \neq 1$$

$$\int \sec^n x dx = \frac{1}{n-1} \sec^{n-2} x \tan x + \frac{n-2}{n-1} \int \sec^{n-2} x dx \text{ if } n \neq 1$$

$$\int \csc^n x dx = \frac{-1}{n-1} \csc^{n-2} x \cot x + \frac{n-2}{n-1} \int \csc^{n-2} x dx \text{ if } n \neq 1$$

$$\int \sin^n x \cos^m x dx = -\frac{\sin^{n-1} x \cos^{m+1} x}{n+m} + \frac{n-1}{n+m} \int \sin^{n-2} x \cos^m x dx \text{ if } n \neq -m$$

$$\int \sin^n x \cos^m x dx = \frac{\sin^{n+1} x \cos^{m-1} x}{n+m} + \frac{m-1}{n+m} \int \sin^n x \cos^{m-2} x dx \text{ if } m \neq -n$$

$$\int x \sin x dx = \sin x - x \cos x + C$$

$$\int x \cos x dx = \cos x + x \sin x + C$$

$$\int x^n \sin x dx = -x^n \cos x + n \int x^{n-1} \cos x dx$$

$$\int x^n \cos x dx = x^n \sin x - n \int x^{n-1} \sin x dx$$